Lux Junior 2009
25. bis 27.9.09 Dörnfeld

A colour space based on advanced colour matching functions

János Schanda, University of Pannonia, Hungary

Abstract

CIE XYZ colour space and colorimetric system served the colour measuring community well for the past 78 years. Never the less already from the 1950's on the attention was drawn on errors of the colour matching functions I(CMFs) of this system. In 1991 CIE established a technical committee to develop a new system of higher precision.

Based on the preliminary results we could show that mismatches observed in visual investigations of LED lights could be drastically reduced if the new system is used.

The use of the new system, based on cone fundamentals, might have advantages in colour rendering investigations.

1. Introduction

CIE based in 1931[1] its 2° Colour Matching Functions (CMFs) on the visual measurements of Guild and Wrigth. The original measurements were performed using real R, G, B primaries, but different ones in the two experiments. The first big victory of the new colorimetric system was, when it turned out that by transforming the two measurement series to a common set of primaries, the two sets provided approximately the same CMFs[2]. In performing the transformation from the RGB primaries to the XYZ primaries[♦] one constrain was to get one of the CMFs equal to the spectral visibility function ($V(\lambda)$), defined in 1924. It soon turned out that this $V(\lambda)$ function is in error, but no official correction of the CIE 1931 colorimetric system was ever introduced. For vision research work later the Vos CMFs[3] became used, see also[4] .

In 1991 CIE formed a new technical committee (CIE TC 1-36) to study existing CMFs, find the most reliable data and propose a chromaticity diagram based on them[5]. In this paper we will summarize the results of this technical committee and show how their findings could be used in practice.

2. The CIE spectral luminous efficiency function

CIE accepted its photometric system in 1924, based on the $V(\lambda)$ function[6]. This conclusion was mainly based on the report by Gibson and Tyndall, who compared their own step-by-step investigations with flicker photometric measurements[7]. Figure 1. is a facsimile reproduction from the original CIE publication showing results from many investigations and the accepted mean curve.

♦ For the fundamentals of CIE Colorimetry see Appendix 1.

CIE was cautious about calling in 1924 this function *relative visibility* without defining what the word *visibility* really means, and recommending it for *provisional* use. Nevertheless since 1924 in all practical photometric and illuminating engineering measurements the photometric quantities have been derived from the radiometric values by integrating the spectral distribution of the radiometric quantity multiplied with the $V(\lambda)$ function over the visible spectrum.

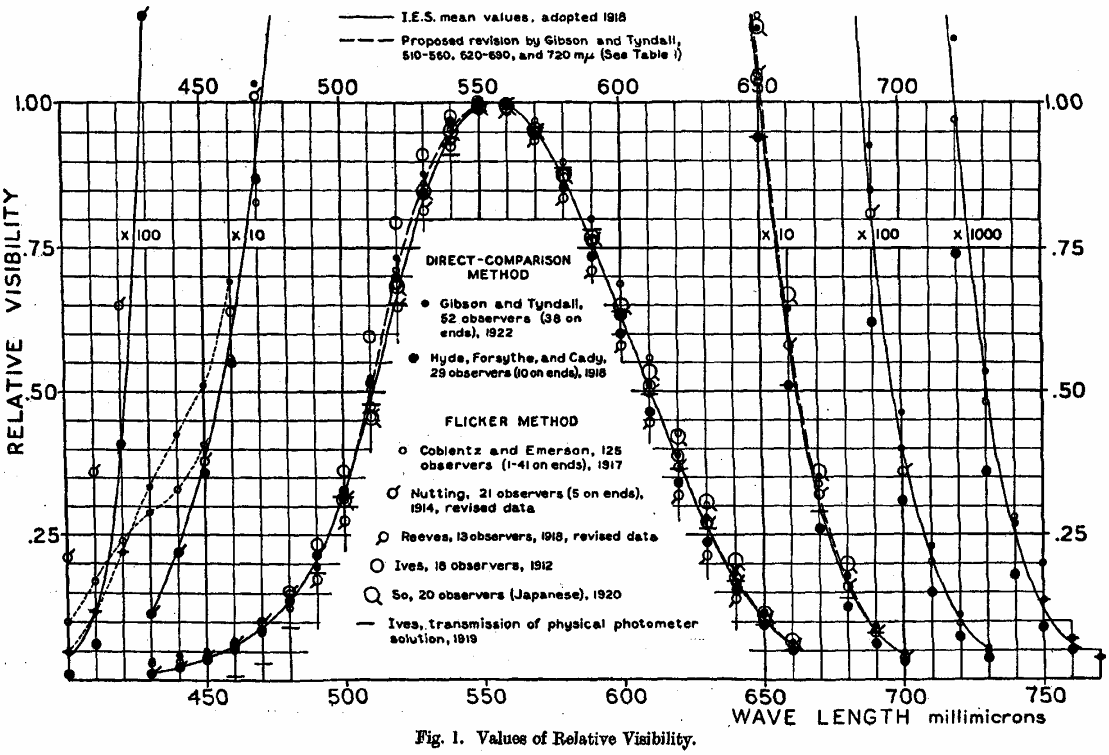

Figure 1. Facsimile reproduction of the CIE 1924 visibility function.

During the past 75 years many investigations have dealt with the question of the visibility function. In 1931, when the CIE colorimetric system was introduced a transformation of the visually determined colour matching functions was made that equated one of the colour matching functions with the $V(\lambda)$ function[1]. It was soon recognised that the data of the original $V(\lambda)$ definition are too low in the blue part of the spectrum. This would have had influence both on the photometric and the colorimetric functions[8]. Nevertheless it was only in 1990 that the CIE officially recommended – as a supplementary function – a modified $V(\lambda)$ function[9], the so called $V_M(\lambda)$ function.

Physiological investigations have shown that that cone signals are responsible for the spectral luminous efficiency function, and that one has to distinguish between brightness perception, where most probably complex interactions between the different cone signals produce the sensation, and a “luminance” perception who’s spectral sensitivity is quite well described by the $V(\lambda)$ function[10], and which is responsible among others for visual acuity, and thus is of primary importance in task lighting.

During the years much speculation took place how the different cone signals feed into the luminance channel. Stockman and Sharpe[11] derived a new spectral luminous efficiency function, $V^*2(\lambda)$, and this became the basis of further research. Figure 2 shows the 1924 $V(\lambda)$-, the $V_M(\lambda)$-, and the $V^*(\lambda)$-function. For practical photometry the luminous flux calculated using the $V_M(\lambda)$ function did not differed much from the standard luminous flux, thus the new system was never adapted in practical photometry.

The situation is different in case of colorimetry, where the fact that the $\bar{y}(\lambda)$ function is identical to the $V(\lambda)$ function produced errors in colorimetry – again small errors in industrial colorimetry, but large enough in ophthalmic research to use different CMFs.

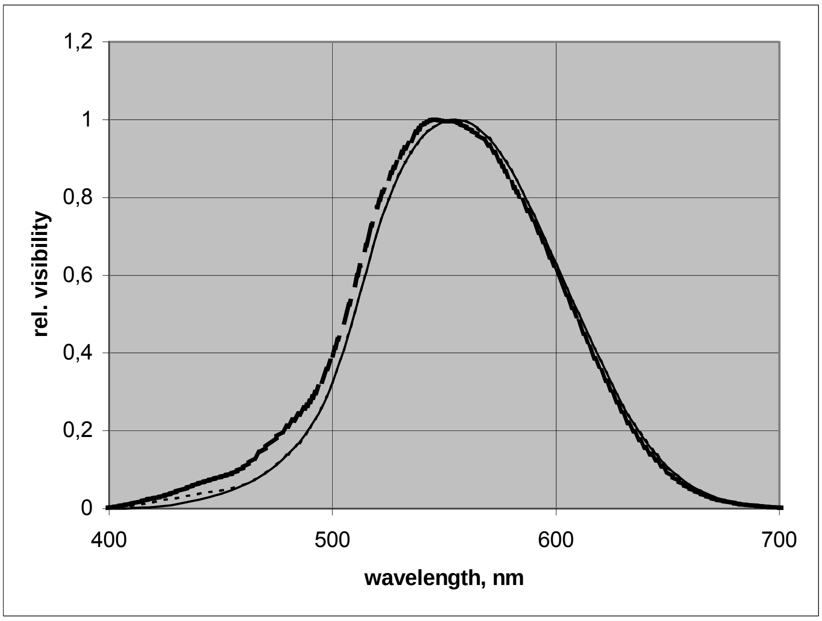

Figure 2. Spectral luminous efficiency (relative visibility) functions: CIE standard $V(\lambda)$-function (thin curve), CIE $V_M(\lambda)$-function (……) and proposed new function by Stockman and Sharpe: $V^*2(\lambda)$ (— — —).

3. CIE work on fundamental colour matching functions

As mentioned in the Introduction in 1991 CIE formed a new technical committee (CIE TC 1-36) to study existing CMFs, find the most reliable data and propose a chromaticity diagram based on them[5].

The committee started from the Stiles and Burch visual CMF data[12,13] as the most reliable ones, and used recent ocular media transmission measurement data to come up first with estimations of cone absorption spectra.

At this point we would like to stress a terminology issue: The cone spectral sensitivity based CMFs at cornea level are called *cone fundamentals*, and the sensitivities at retinal level (i.e. taking ocular media transmission into consideration) are called cone excitations. From the Stiles-Burch RGB CMFs the cone fundamentals of their observers could be determined. By measuring the transmission of the lens and other preretinal media, which are partly age dependent, partly vary with location in the eye, one can get to the retinal level. The macular pigment transmission optical density determination is a very complex tsk, as concentration dependence, light path in the cone, etc. has to be considered.

A further – fundamental – decision was, to reach to the cone absorption spectra, how to get from the Stiles-Burch CMFs to real cone excitations. As we know - and seen to get from the RGB space to the XYZ space – an infinite number of spectral sensitivity triads could be constructed. One further constrain is needed to get to real cone excitations. This was the so called König hypothesis that the cone absorption spectra of dichromats is the same as the two corresponding cone absorption spectra of a trichromat.

By the help of these data one could determine the cone photopigment absorption spectra, and these should be already age independent if the proper age related absorptions were used in getting from the Stiles-Burch data to the cone excitations.

Figure ***3*** shows the visual pigment absorption spectra.

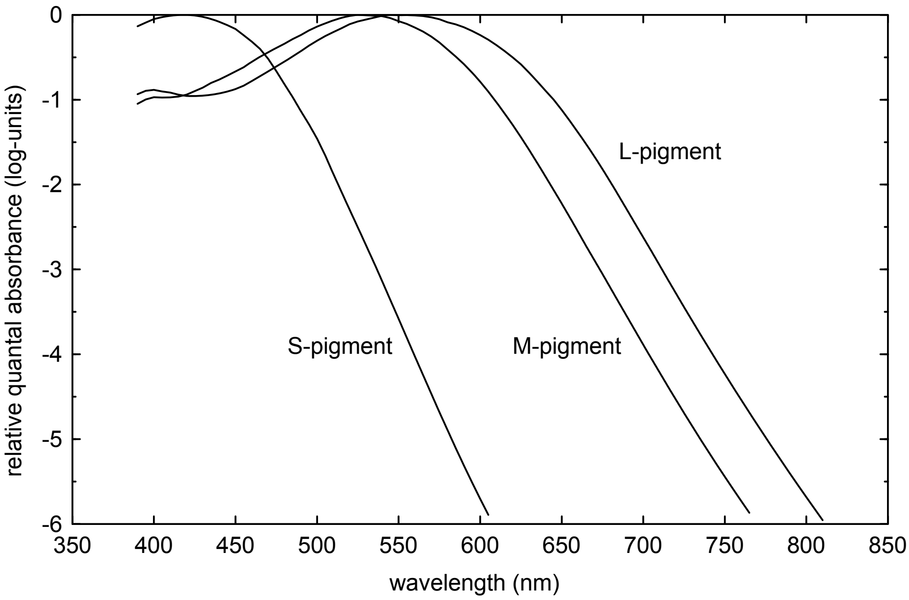

Figure 3. The low density absorbance spectra of the visual pigments in terms of quanta.

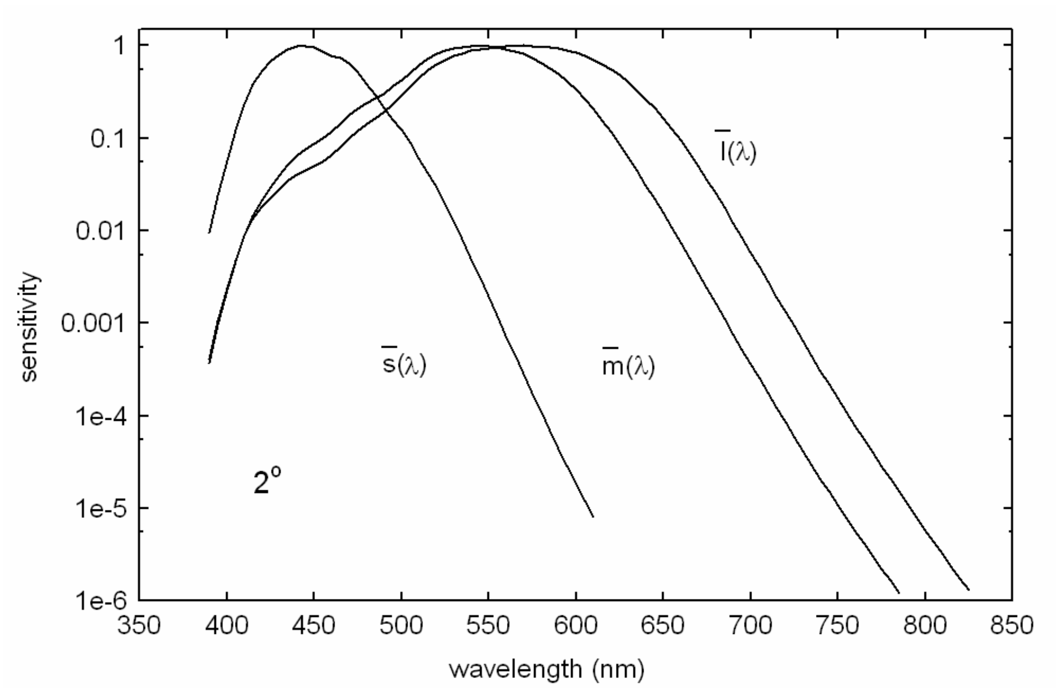

Figure 4. The cone fundamentals for 2º viewing field

Knowing the average age dependence of the different ocular media and the filed size, one can determine for any age and filed size the cone fundamentals (i.e. fundamental CMFs). *Figure 4* shos e.g. the cone fundamentals for a 10°field, young observer. Final decisions on some minor questions are still conducted in CIE TC 1-36, the actual cone fundamentals can be found on the Internet at[14].

4. Transformation into practical colour matching functions

From the LMS cone fundamentals one can get to an XYZ-like space with a similar matrix transformation as was used to get from the RGB space to the XYZ space. A draft report of CIE TC 1-36 suggests the following transformation:

$$\begin{vmatrix} \bar{x}_{\mathrm{F}}(\lambda) \\ \bar{y}_{\mathrm{F}}(\lambda) \\ \bar{z}_{\mathrm{F}}(\lambda) \end{vmatrix} = \begin{vmatrix} 1.910988 & 1.394658 & 0.389317 \\ 0.643151 & 0.395946 & 0.000000 \\ 0.000000 & 0.000000 & 1.919339 \end{vmatrix} \bullet \begin{vmatrix} \bar{l}(\lambda) \\ \bar{m}(\lambda \\ \bar{s}(\lambda) \end{vmatrix} \qquad (1)$$

Using this transformation one gets the cone fundamental related CMFs in an XYZ like space. Figure 5 shows as an example the standard and cone fundamental 2° CMFs.

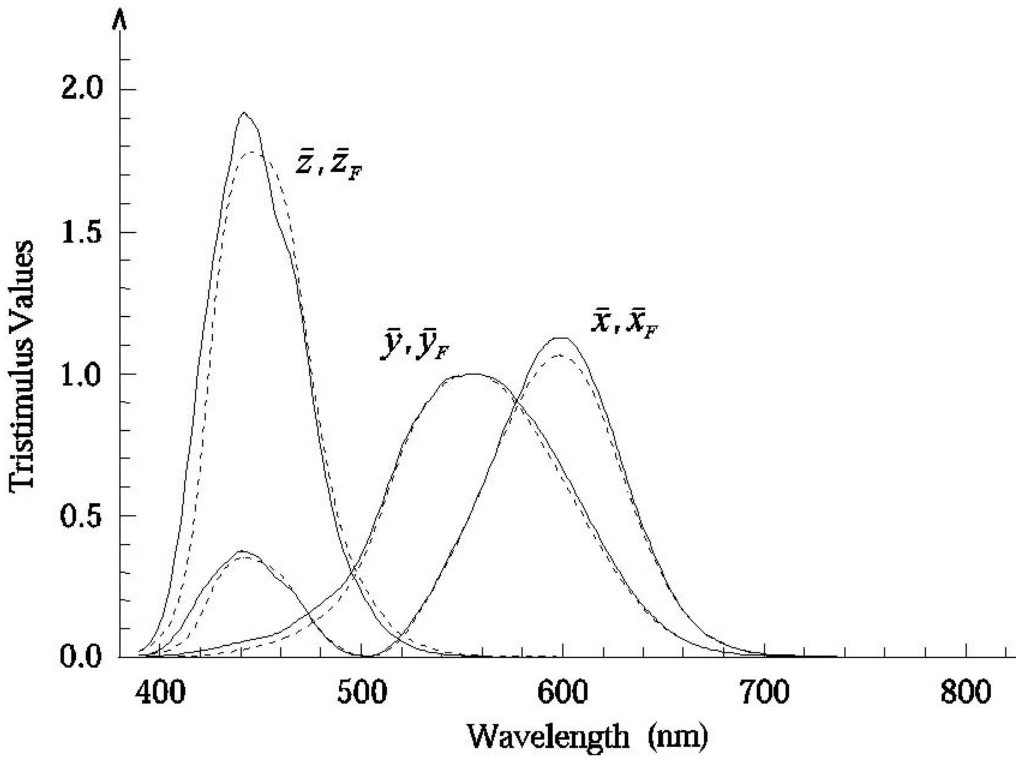

Figure 5. CIE 2° (dashed curves) and cone fundamental derived (full curves) 2° CMFs.

The chromaticity diagram based on these CMFs is seen in Figure 6. The inlet in this figure shows the size of the difference along the spectrum locus. As can be seen in some parts of the chromaticity diagram the difference is considerable. This prompted us to look how large differences might occur if one tries to use this colour space instead of the CIE 1931 space to describe properties of some modern light sources, e.g. LEDs.

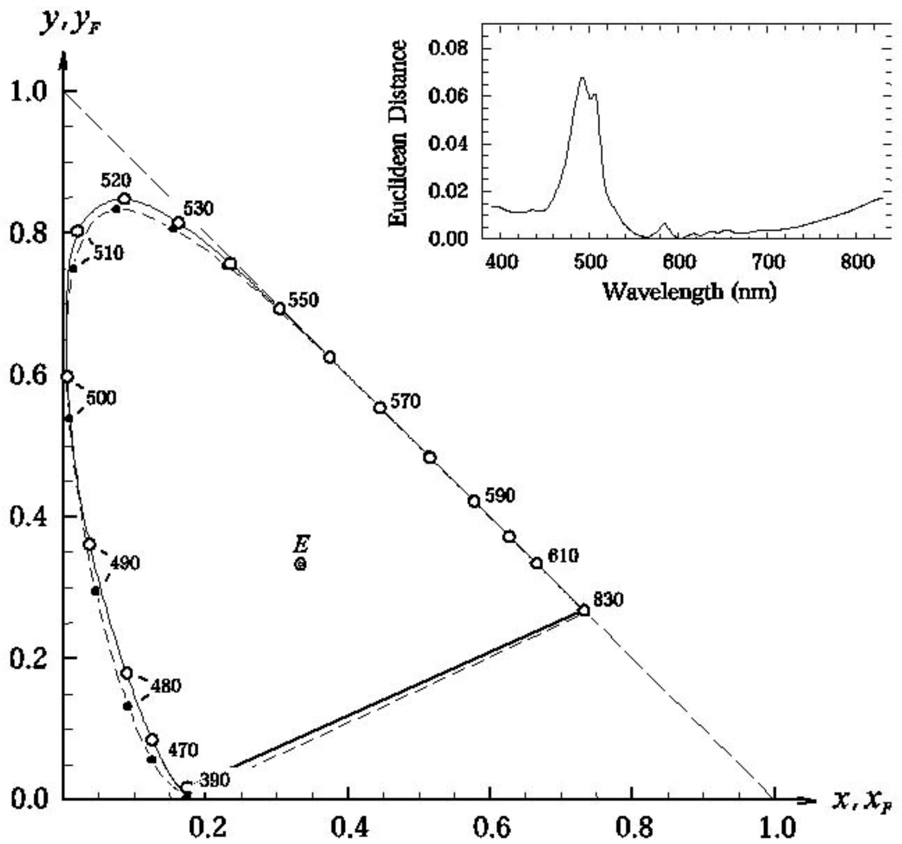

Figure 6. Standard and cone fundamental chromaticity diagram. The inlet shows the Euclidean distance between the corresponding points on the spectrum locus.

5. Tests of the cone fundamental based CMFs

At the end of the 20th century Thornton reported in his three part paper[15] highly different errors between the instrumental and visual matches if he changed the primaries for the visual matches from what he termed “prime colour” wavelengths to his “non-prime” or “anti-prime” wavelengths.

In recent years several papers dealt with the question of the validity of Grassmann’s laws, and the transformability of primaries (see e.g.[16,17]), and whether this could help to understand Thornton’s findings. The present paper does not wish to question the validity of those papers, it just would like to show what effect a change of the CIE 1931 colorimetric system to the system based on cone fundamentals (or similar) could have for the colorimetry of lights produced by the additive mixture of the light of red, green and blue LEDs of different dominant wavelengths.

We were also interested whether using better CMFs could explain some of Thornton’s observations when his different primaries were used[18]. Error! Reference source not found. shows the wavelengths of Thornton’s Prime (PC), Non-Prime (NP) and Anti-Prime (AP) primary colours and also the dominant wavelengths of the LED primaries used in our experiments. We had RGB LED clusters with dominant wavelengths near the Thornton PC (Exp. ‘A’ and ‘B’) and AP (Exp ‘C’) primary groups.

Table 1: Thornton primary groups (PC, NP, AP) locations and dominant wavelengths [nm] of the RGB LED primaries used in our experiments (‘A’,’B’ and ‘C’)

Thornton’s primary groups	R	G	B
PC	610	530	450
NP	640	560	480
AP	650	580	500
Experiment ID	R-LED	G-LED	B-LED
'A'	626	523	462
'B'	626	525	473
'C'	639	593	507

Thornton performed his measurements using Maxwell matches, comparing two near white visual fields. For the set of PC primaries Thornton got good agreement between visual matches and instrumental matches. As he changed from the PC primaries to the NP and AP primaries the errors between the visual and instrumental matches increased. As the PC primaries are nearer to the primaries used to develop the CIE 1931 colorimetric system[19] one could have expected that these should work well. Some of the following results were discussed in the joint paper with Ms. Bieske, Ilmenau[20] , where instrumental colour differences up to 10 ΔE_{ab} could be observed for visual matches between low chroma lights produced by mixing the light of red, green and blue LEDs and filtered incandescent light, and these could be halved by using the cone fundamental derived CMFs [21,22] .

5.1 General set-up

All experiments introduced here where basic colour matching set-ups using two matching small angle (2° – 3°) fields (reference field and test field). The observers had the task to match the chromaticity of the test field to the chromaticity of the reference field. The observers repeated the matches several times (usually ten times) and after each match the spectral power distribution of the stimuli (reference and test) was measured using a well calibrated spectroradiometer. During the evaluation these measurement data were used to calculate the chromaticity using different colour matching function sets (e.g. CIE 1931 2° CMFs, Fundamental CMFs…).

The main differences between the experiments introduced in this paper are the followings: in experiment ‘A’ the users did match white reflecting fields using PC-like primaries as test source; in experiment ‘B’ the users had to match nine samples of more saturated self luminous reference stimuli using PC-like primaries as test source; in experiment ‘C’ only one coloured sample was presented as a reference and the users had AP-like primaries as test source.

5.2 Experiment ‘A’

The first experiment was a Maxwellian-like colour matching experiment carried out using white reflecting references at two different correlated colour temperatures (warm white ~2 850 K, cool white ~6 500 K). The experiment using a cool white illuminant reference was carried out at the TU (Technical University) Ilmenau in Germany, while the warm white reference experiments were carried out at the UP (University of Pannonia) in Veszprém, Hungary. The sources used to illuminate the references was a halogen incandescent lamp for a warm white reference and a HMI lamp+blue filter combination for the cool white reference.

To achieve a colour match the users could change the channel currents of the LEDs in the RGB cluster. The results are summarized in Figure **7**. We can see that one could have a better match in terms of the calculated chromaticities when one used the Fundamental CMFs.

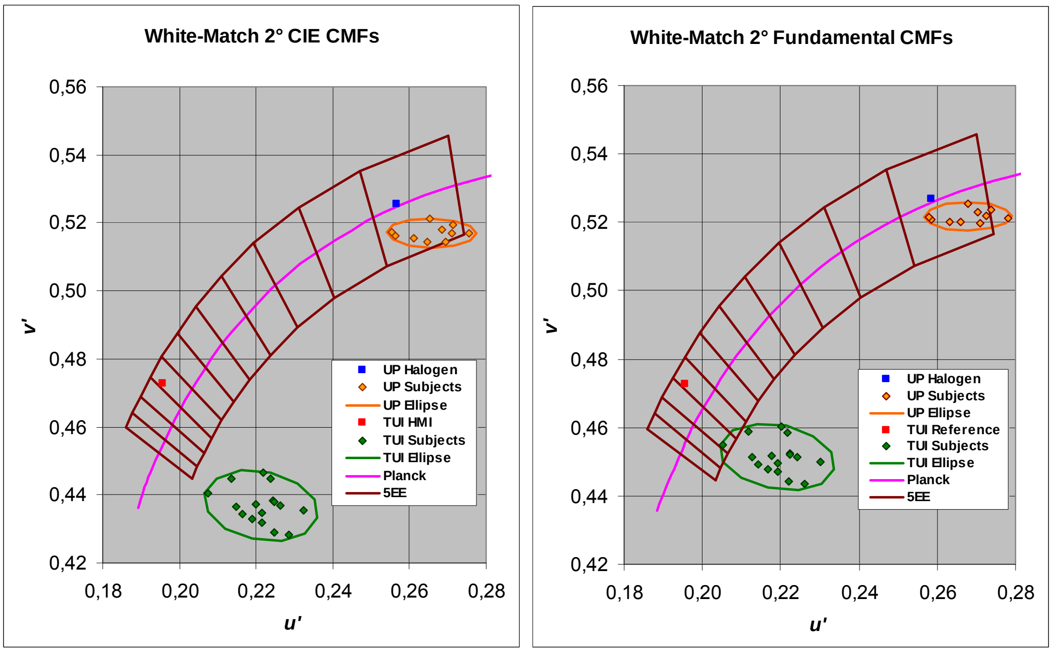

Figure 7**.** Comparing the results of experiment 'A', the chromaticity differences between the references and the average of the observers could be decreased by ~40% if the fundamental CMFs are used.

5.3 Experiment 'B'

In Experiment 'B' tests were made in several parts of the chromaticity diagram. Figure 8 shows the nine test points △shows the chromaticity of the filtered incandescent light, ○ show chromaticities of measured for RGB LEDs of visual match. In this experiment we could halve the chromatic differences if we used the Fundamental CMFs.

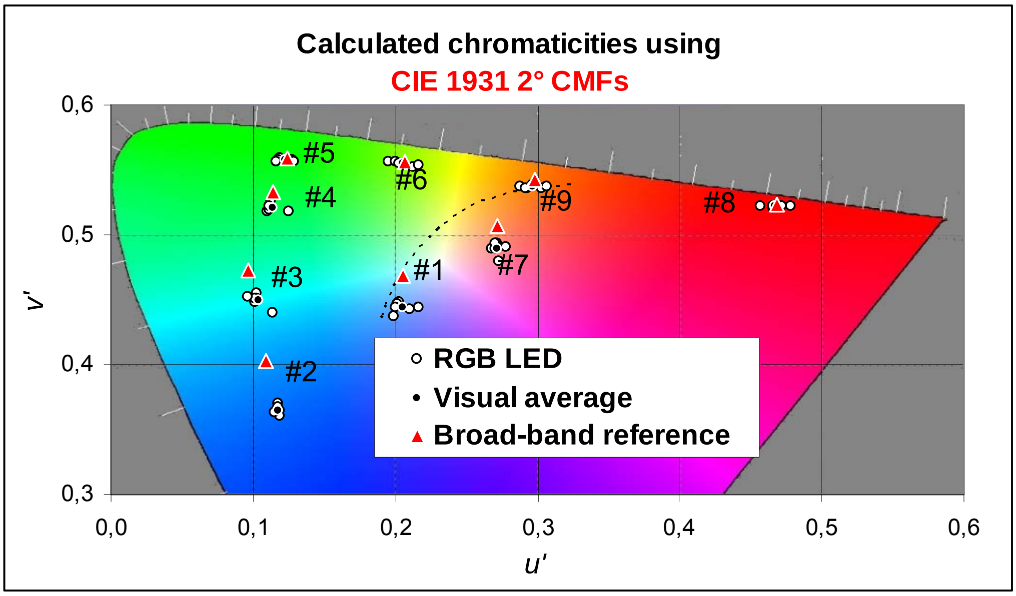

Figure 8. The nine (#1 – #9) colour references used in experiment 'B' with chromaticities of visually matching LED chromaticities.

5.4 Experiment 'C'

The basic idea for experiment 'C' was to check if the change of the primaries from PC-like ones to AP-like ones (see Error! Reference source not found.) changes or not the Thornton findings mentioned in the introduction. For this experiment the reference point #2 and the Experiment ID 'C' LEDs were used, by matching #2 + G-LED with the light of R-LED + B-LED. Also in this case it turned out that the colorimetric mismatch for the visual match could be halved by using the CIE TC 1-36 proposed fundamental based CMFs.

6. Practical applications

The observed differences between colorimetric and visual matches might have an influence on other colorimetric properties, as e.g. chromaticity description of LEDs and colour rendering. Table 2 shows chromaticity co-ordinates of some LEDs if these are calculated using the standard CIE 2°observer, and the cone fundamental based CMFs. A can be seen for the blue and green LEDs the differences are non-negligible

Table 2: Chromaticity co-ordinates of some LEDs using the CIE 1931 CMFs and the CIE TC 1-36 fundamental CMFs

	x	y	x_F	y_F
White1	0.314	0.319	0.320	0.331
White2	0.307	0.330	0.311	0.337
White3	0.305	0.306	0.309	0.313
Blue	0.149	0.031	0.148	0.046
Green	0.276	0.695	0.282	0.699
Orange	0.687	0.313	0.686	0.315
Red	0.686	0.314	0.685	0.315

In calculating the colour rendering index one compares the chromaticity of the sample illuminated once by a continuous light (incandescent or daylight), and than by the test light source, thus e.g. by an LED. We could show that the instrumental chromaticity difference in case of visual colour match is well observable. Thus the question arises whether this has an influence on the calculated colour rendering indices. Figure 9 shows the relative spectral power distribution of an RGB-LED of 4200 K correlated colour temperature. We have calculated the colour rendering indices for this LED using the standard method, and changing the CMFs to the cone fundamental based CMFs. Table 3 compares the two sets of Ri-s and the Ra values. As can be seen the difference is not too big, but for some samples it is non-negligible.

Naturally to get to a better description of colour rendering, other aspects of the calculation method have to be updated as well[23]. Colour spaces based on a colour appearance model seem to be better suited to describe colour quality of light sources, but also in those cases the use of an updated set of CMFs can be recommended.

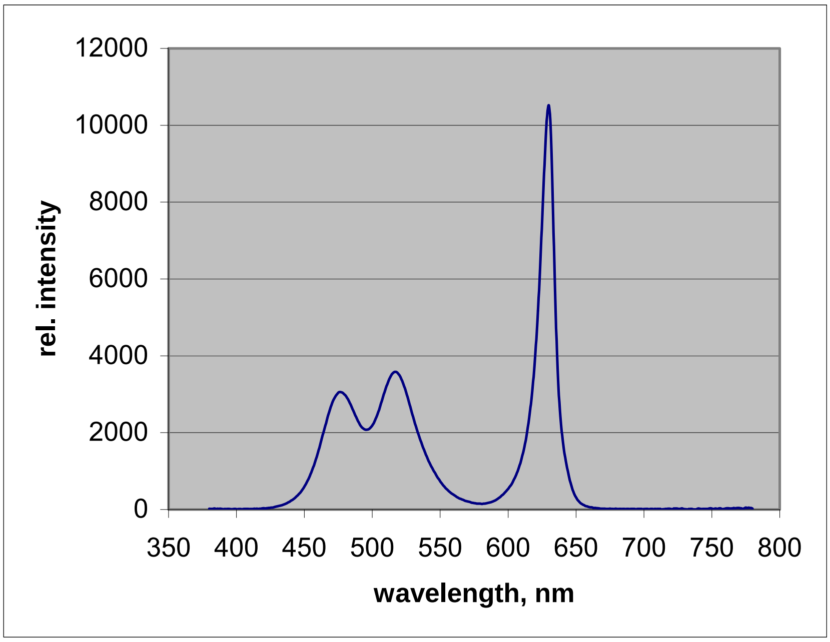

Figure 9. Spectral power distribution of an RGB-LED of approximately 4200 K.

Table 3. Special and general colour rendering indices of an RGB-LED, using the standard and cone fundamental based CMFs

R1	-4.68	0.51	R9	-191.73	-177.64
R2	40.35	42.61	R10	-41.23	-37.82
R3	70.51	71.02	R11	-17.26	-11.36
R4	3.49	7.55	R12	0.32	6.53
R5	6.43	10.82	R13	3.31	8.18
R6	14.85	19.11	R14	81.64	82.17
R7	47.14	48.00			
R8	-32.24	-27.31	Ra	18.23	21.54

7. Summary and conclusions

Development colour spaces has a long history. The CIE 1931 colorimetric system got general acceptance and many industries use it, despite the fact that it is well known that the CMFs on which it is based are in error. Only very recently, with the introduction of LEDs it became necessary to consider eventual update of the present system.

The now 50 year old colour matching experiments of Stiles and Burch seem still to be the most accurate determination of the average human colour matching functions. Based on these, but taking more recent data on ocular medium transmission characteristics CIE TC 1-36 came up with a set of cone fundamentals. Based on their results one can calculate observer age and field size dependent CMFs – or can select a set for a given task.

Calculations have been performed to transform these cone fundamentals into CMFs that resemble those of the CIE XYZ colour matching functions. Using these – still not finally accepted – CMFs calculations have been performed that provided better agreement between the colorimetric matches and visual observations of highly metameric test stimuli, as e.g. of matching white light produced by RGB-LEDs and incandescent light.

Experiments are under way to prove even better agreements between visual and instrumental matches.

Appendix 1.

CIE standard colorimetric observers[24]

Basic colorimetry, the description of the results of colour matching experiments, is built on additive colour mixing, because the laws of additive colour mixing are simpler then those of subtractive colour mixing. The basic empirical laws of additive colour mixing were formulated in 1853 by HG Grassmann[25], reproduced here in its more modern form[26]:

1. To specify a colour match, three independent variables are necessary and sufficient.
2. For an additive mixture of colour stimuli, only their tristimulus values are relevant, not their spectral compositions.
3. In additive mixtures of colour stimuli, if one or more components of the mixture are gradually changed, the resulting tristimulus values also change gradually.

CIE colorimetry[27] builds on these empirical laws that hold reasonably well as long as the observation conditions (e.g. size of stimuli, presentation on the retina: foveal or parafoveal, etc), previous exposure of the observer's eye, and the person who makes the matching are kept the same. Therefore the observation conditions have been standardized: foveal vision, 2° or 10° field size, dark surrounding; as previous exposure a sufficiently long dark adaptation is supposed and the standardized colour matching functions have been determined by averaging the results of a large number of observers. For questions relating to the validity of Grassmann's laws see[28].

According to Grassman's laws a colour stimulus can be matched by the additive mixture of three properly selected stimuli (properly selected includes independent, i.e. none of the stimuli can be matched by the additive mixture of the other two stimuli). Figure A1-1 shows the basic experiment of obtaining a colour match. The test stimulus is projected on one side of a bipartite field, the additive mixture of the three matching stimuli (it is practical to use monochromatic Red, Green and Blue lights, see later) is projected onto the other side of the field. By using adjustable light attenuators, the light flux of the three matching stimuli are adjusted to obtain a colour appearance match between the two fields. When this situation is reached the test stimulus can be characterized by the three luminance values of the matching stimuli reaching the eye of the observer.

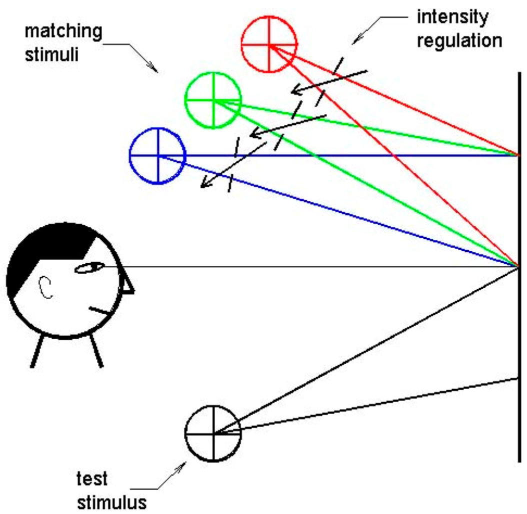

Figure A1-1: Basic experiment of colour matching.

The spectral power distributions of the test stimulus and of the additive mixture of the three matching stimuli are usually different. In such cases we speak about metameric colours: they look nearly alike to the human observer[♦] (having equal tristimulus values, see later), but their spectral power distribution is different. Metamerism is fundamental in colorimetry.

To obtain a colorimetric system one has to define the matching stimuli, specifying both their spectral composition and the units in which their amounts are measured. If this is done one can describe a colour match in the following form:

$$[C] \equiv R[R] + G[G] + B[B] \qquad \text{A1-1}$$

where [C] is the unknown stimulus; "≡" reads as "matches"; [R], [G], [B] are the units of the matching stimuli and *R, G, B* represent the amounts to be used, expressed in the adopted units, of the matching stimuli to reach a match.

As a next step one has to determine for every monochromatic constituent of the equi-energy spectrum (the spectrum having equal power per small constant wavelength intervals throughout the visible spectrum) the amounts of the three matching stimuli needed to achieve a match. The wavelength dependent amounts needed for the above colour match of the monochromatic test stimuli are called colour matching functions and are written in the following form: $\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)$. Because of the additivity and multiplicativity of colour stimuli, for a non-monochromatic test colour stimulus, $P(\lambda)$, the amounts of the matching stimuli needed for a match can be determined by adding the amounts needed to match the monochromatic components of the test stimulus (for a detailed analysis see e.g. [29]):

$$[C] = \int_{380nm}^{780nm} \bar{r}(\lambda)P(\lambda)d\lambda \cdot [R] + \int_{380nm}^{780nm} \bar{g}(\lambda)P(\lambda)d\lambda \cdot [G] + \int_{380nm}^{780nm} \bar{b}(\lambda)P(\lambda)d\lambda \cdot [B] \qquad \text{(A1-2)}$$

The $\int_{380nm}^{780nm} \bar{r}(\lambda)P(\lambda)d\lambda$, $\int_{380nm}^{780nm} \bar{g}(\lambda)P(\lambda)d\lambda$, $\int_{380nm}^{780nm} \bar{b}(\lambda)P(\lambda)d\lambda$ integrals are called tristimulus values and can serve as the descriptors of the colour stimulus and according to Equation (A1-1) the symbols *R, G, B* are used.

To be able to define a standard observer the spectral compositions and the luminances of the primaries have to be specified. Single wavelengths were used: 700 nm for the Red, 546.1 nm for the Green and 435.8 nm for the Blue primary. To these primaries the data obtained by Guild[19] and Wright[30,31] have been transformed. The "unit intensity" of the primaries was defined by stating their luminances. The requirement was that for an equi-energy spectrum the addition of the unit amounts of the three primaries should give a colour match. If 1 cd/m^2 of Red light was used, then 4.5907 cd/m^2 of Green and 0.0601 cd/m^2 Blue light was needed to match the colour of an equi-energy spectrum.

Performing colour matches using these matching stimuli one gets the colour matching functions (CMFs) depicted in Figure A1-2. The negative lobes in these curves refer to the fact that in some parts of the spectrum a match can be obtained only if one of the matching stimuli is added to the test stimulus.

As mentioned the units of the three primaries have been defined by their luminances and thus the luminance of a colour stimulus with the tristimulus values of *R*, *G*, *B* will be:

$$L = 1.0000R + 4.5907G + 0.0601B \qquad \text{A1-3}$$

♦ In the main text of the paper the problem of metamerism will be discussed in some detail.

But the units used are very often only defined as relative luminances, so that *L* is in these cases only a relative luminance.

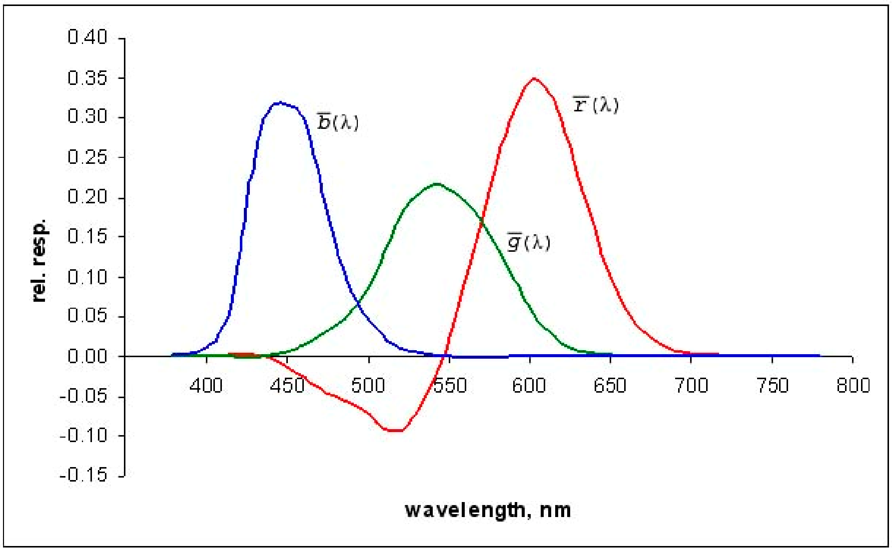

Figure A1-2: $\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)$ *CMFs of the CIE 1931 standard colorimetric observer.*

In many colorimetric calculations – especially at the time of standardizing the trichromatic system, when no computers were available – the negative lobes in the CMFs made calculations more difficult, therefore in 1931 the CIE decided to transform from the real [R], [G], [B] primaries to a set of imaginary primaries [X], [Y], [Z], where the CMFs have no negative lobes. Further requirements were that the tristimulus values of an equi-energy stimulus should be equal ($X = Y = Z$), that one of the tristimulus values should provide photometric quantities (thus one of the CMFs should be equal to the $V(\lambda)$ function), and that the volume of the tetrahedron set by the new primaries should be as small as possible.

Based on above requirements one gets the following matrix transformation between the *R, G, B* and the new *X, Y, Z* tristimulus values:

$$\begin{vmatrix} X \\ Y \\ Z \end{vmatrix} = \begin{vmatrix} 2.768\,892 & 1.751\,748 & 1.130\,160 \\ 1.000\,000 & 4.590\,700 & 0.060\,100 \\ 0 & 0.056\,508 & 5.594\,292 \end{vmatrix} \bullet \begin{vmatrix} R \\ G \\ B \end{vmatrix} \qquad \text{(A1-4)}$$

As can be seen the *Y* tristimulus value will add up to a (relative) photometric quantity as defined in Equation (A1-3). The CMFs are the tristimulus values of monochromatic radiations, thus the $\bar{x}(\lambda)$, $\bar{y}(\lambda) = V(\lambda)$, $\bar{z}(\lambda)$ functions can be calculated from the $\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)$ CMFs using the above equation.

Figure A1-3 shows the colour matching functions (CMFs) of the CIE 1931 standard colorimetric observer. This observer should be used if the fields to be matched subtend between about 1° and about 4° at the eye of the observer. In technical applications this observer is often written as 2°-standard colorimetric observer. (A 2° visual field represents a diameter of about 17 mm at a viewing distance of 0.5 m.). As this central part of the retina, the fovea is covered by a yellow pigmented disc, the macula lutea, the colour sensitivity of the eye differs in this central part from the colour sensitivity of the adjacent regions. In 1964 CIE standardized CMFs for a 10°observation field, the symbols of the CMFs for this large filed are $\bar{x}_{10}(\lambda), \bar{y}_{10}(\lambda), \bar{z}_{10}(\lambda)$**,** and shown in Figure A1-3 by crosses (***x*).**

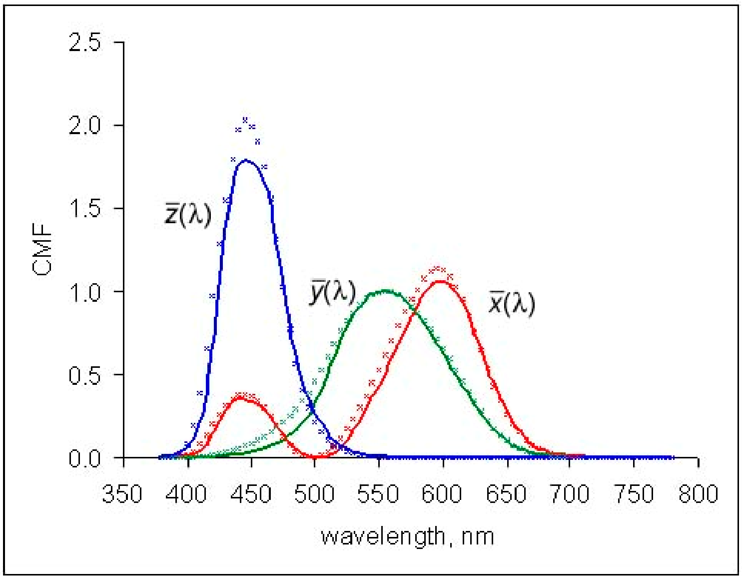

Figure A1-3: The $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ colour matching functions of the CIE 1931 standard (2°) colorimetric observer and, the $\bar{x}_{10}(\lambda), \bar{y}_{10}(\lambda), \bar{z}_{10}(\lambda)$ CMFs of the CIE 1964 standard observer shown by …x… .

Values of the CIE 1931 standard colorimetric observer have been standardized[32,33].

As mentioned in connection with Equation (A1-2) the amounts of the primaries to achieve a match are called tristimulus values. In the case of the CIE-XYZ trichromatic system the tristimulus values are defined as

$$X = k\int_{380nm,}^{780nm}\phi_\lambda(\lambda)\bar{x}(\lambda)\mathrm{d}\lambda\,,\; Y = k\int_{380nm,}^{780nm}\phi_\lambda(\lambda)\bar{y}(\lambda)\mathrm{d}\lambda\,,\; Z = k\int_{380nm,}^{780nm}\phi_\lambda(\lambda)\bar{z}(\lambda)\mathrm{d}\lambda \qquad \text{(A1-5)}$$

where $\phi_\lambda(\lambda)$ is the colour stimulus function of the light seen by the observer,
k is a constant; for self-luminous objects one uses *k*= 683 lm/W to get to photometric quantities, and
$\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ are the colour matching functions (CMF) of the CIE 1931 standard observer.

According to the CIE recommendation[27] the integration can be carried out by numerical summation at wavelength intervals, $\Delta\lambda$, equal to 1 nm:

$$\begin{aligned} X &= k\sum_\lambda \phi_\lambda(\lambda)\bar{x}(\lambda)\Delta\lambda \\ Y &= k\sum_\lambda \phi_\lambda(\lambda)\bar{y}(\lambda)\Delta\lambda \\ Z &= k\sum_\lambda \phi_\lambda(\lambda)\bar{z}(\lambda)\Delta\lambda \end{aligned} \qquad \text{(3 - 6)}$$

References

1 CIE (1932) Colorimétrie, Resolutions 1 – 4. Recueil des travaux et compte rendu des séances, Hutième Session Cambridge – Septembre 1931. Publ.: Bureau Central de la Commission, The National Physical Laboratory Teddington, Cambridge at the University Press. pp. 19-29.

2 See Wright, WD (1981) The historical and experimental background to the 1931 CIE system of colorimetry in Golden Jubilee of Colour in CIE, The Soc. Of Dyers and Colourists, Bradford, reproduced also in Colorimetry, Understanding the CIE system, ed. J. Schanda, pp. 9-23. Wiley 2007.

3 Vos JJ (1978) Colorimetric and photometric properties of a 2° fundamental observer, *COLOR Res. & Appl*. **3**/3 125-128.

4 Vos JJ, Estévez O & Walraven PL (1990) Improved colour fundamentals offer a new view on photometric additivity. *Vision Research,* **30** 937-943.

5 Viénot F, Walraven P (2007) Colour-mathching functions, Physiological basis, in Colorimetry, Understanding the CIE System, ed. J. Schanda, pp. 219-243. Wiley 2007.

6 Gibson KS, The relative visibility function, CIE Sixième Session, Genève, Juillet, 1924. Recueil des Travaux et Compte Rendu de Séances, Cambridge, the Univ. Press, 1926, pp. 232-238.

7 Gibson KS and Tyndall EPT, Visibility of radiant energy, *Bureau of Standards Scientific Papers*, No. 475; 131-191 1923.

8 ICI TC 7, Colorimetry and artificial daylight, Report of Secretariat, *CIE Douzième Session*, Stockholm, 1951, Receuil des travaux et compte rendu des séances, Vol. **1**. 7.p1-60.

9 Commission Internationale de l'Éclairage, 1988 2° spectral luminous efficiency function for photopic vision, **CIE** *86*-1990

10 Lennie P; Pokorny J; Smith V C (1993) Luminance. *J of Opt. Soc. Am.*, **10**/6, 1283-1293.

11 Stockman A, Sharpe LT, The spectral sensitivity of the middle- and long-wavelength-sensitive cones derived from measurements in observers of known genotype. Vision Res. **40** 1711-1737 2000.

12 Stiles WS, Burch JM (1955) Interim report to the Commission International de l'Eclairage, Zurich, 1955, on the National Physical Laboratory's investigation of colour-matching. *Optica Acta* **2** 168-181.

13 Stiles WS & Burch JM (1959) NPL colour-matching investigation: Final report (1958). *Optica Acta*, **6** 1-26.

14 http://www.cvrl.org/

15 Thornton WA (1992) Towards a more accurate and extensible colorimetry. Part 1. *Color Res. & Appl.* **17** 79-122; Part 2. *Color Res. & Appl.* **17** 162-186; Part 3. *Color Res. & Appl.* **17** 240-262.

16 Brill MH (2006) Towards resolving open questions on the validity of Grassmann's laws. *Proc. ISCC/CIE Expert Symp. '06 – 75 years of the CIE standard colorimetric observer*. Publ. **CIE x03**0:2006. pp.8-12.

17 Oicherman B, Luo MR, Robertson AR (2006) Test of the transformation of tristimulus space: Forward- and inverse-matrix methods. *Proc. ISCC/CIE Expert Symp. '06 – 75 years of the CIE standard colorimetric observer*. Publ. **CIE x030**:2006. pp. 30-36.

18 Csuti P, Schanda J (2009) A better description of metameric experience of LED clusters. Light & Lighting Conf. Budapest.

19 Guild, J (1931) The colorimetric properties of the spectrum, *Phil. Trans. Roy. Soc. Lond.*, Ser. A, **230**, 149-187.

20 Bieske K, Csuti P, Schanda J: "Colour Appearance of Metameric Lights and Possible Colorimetric Description", *Poster on the CIE Expert Symposium on Appearance*, Paris, France, Oct 2006, CIE x032:2007.

21 CIE Techn. Report: Fundamental Chromaticity Diagram with Physiological Axes - Part 1 *Publ.* **CIE 170-1:**2006.

22 *CIE TC 1-36 draft technical report of Chapter 7.3*: Development of chromaticity diagrams based upon the principles of the CIE XYZ system (draft 2006).

23 Sándor N, Schanda J (2006) Visual colour rendering based on colour difference evaluations. Lighting Res. & Technol. **38**/3 225-239.

24 From Schanda, J (2007) CIE colorimery, in Colorimetry, Understanding the CIE system, ed. J. Schanda, pp. 25-78. Wiley 2007.

25 Grassmann HG (1853) Zur Theorie der Farbenmischung/Theory of compound colours. Original published in *Poggendorf' Ann. Phys.*,**89** 69 original translation in English in *Phil. Mag.* 4(7) 254-264 1854; present formulation from *Sources of Color Science* 53-60 MIT Press 1970; see Selected papers in Colorimetry – Fundamentals, ed.: MacAdam DL, SPIE Milestone Series MS 77 1993 pp.10-13.

26 Hunt RWG (1998) Measuring colour, third edition. Fountain Press, England.

27 CIE Technical Report (2004) Colorimetry, 3rd ed. Publication 15:2004, CIE Central Bureau, Vienna.

28 Brill, MH, Robertson AR (2007) Open problems on the validity of Grassmann's laws in Colorimetry, Understanding the CIE system, ed. J. Schanda, pp. 245-259. Wiley 2007.

29 Schanda JD (1997) Colorimetry, in Handbook of Applied Photometry, ed. DeCusatis C,AIP Press, Woodbury, NY 1997 pp.327-412.

30 Wright WD (1928-29) A re-determination of the trichromatic coefficients of the spectral colours. *Trans. Opt. Soc. Lond.*, **30** 141-164.

31 Wright WD (1929-30) A re-determination of the mixture curves of the spectrum. *Trans. Opt. Soc. Lond.*, **31** 201-218.

32 ISO/CIE 10527(E) joint ISO/CIE standard: Colorimetric observers, 1991 (S002, 1986).

33 CIE Draft Standard DS 014-1.2/E:2004: Colorimetry - Part 1: CIE Standard Colorimetric Observers.